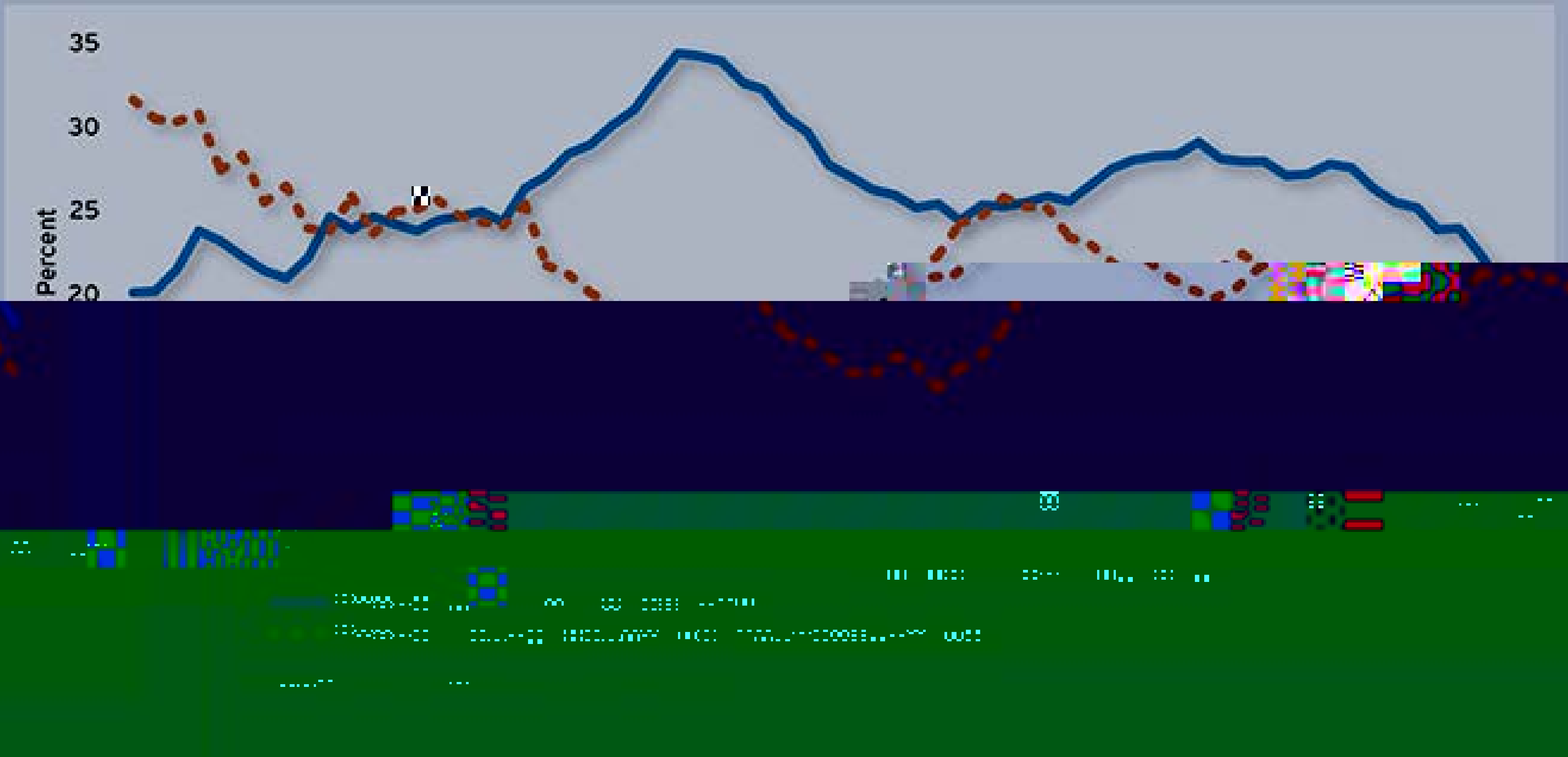
Math Slide (Figure 1)

The share of American males studying math-intensive subjects has fallen; it declined dramatically when the share completing college rose, but recently has slid downward along with college completion rates.



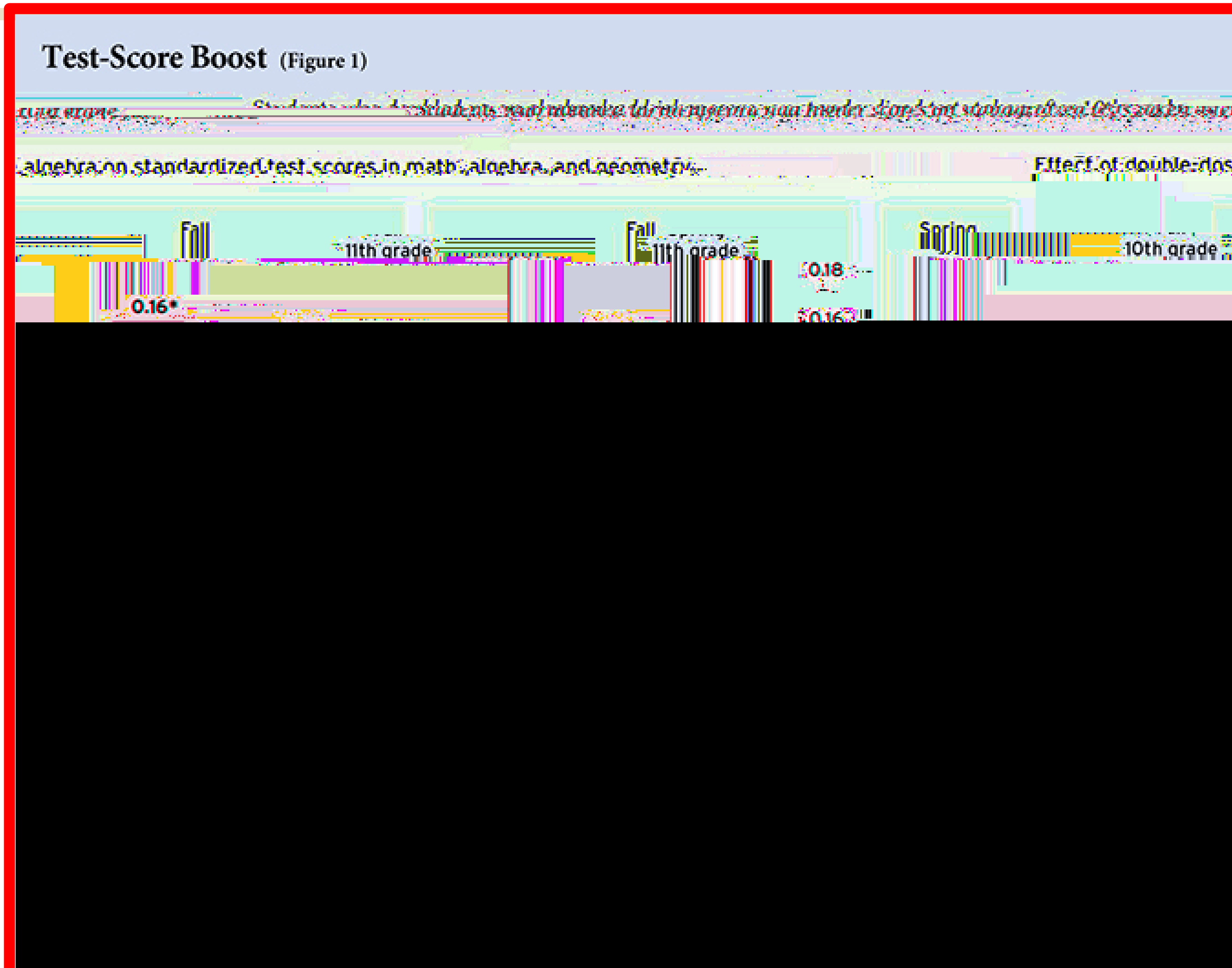
Research on Algebraic Understanding

- Recent research suggests that inappropriate instruction may have had a decisive role in the poor results from early studies of algebraic reasoning among adolescents (Booth, 1988; Schliemann & Carraher, 2002).

- Studies of systemic algebra instruction have provided equivocal findings (Clotfelter, Ladd, &Vigdor, 2012; Cortes, Goodman, & Nomi, 2013)

Effects of Accelerating Algebra

Impact of Double-Doses of Algebra



Reading and Writing in Algebra (cont'd)

ing skills benefited more from the algebra
wise similar students, perhaps because reading
al to the instructional model.

Students with weak read
support class than other
and writing were centr

Critical Topics for Teaching and Learning Algebra

(1) Variables and constants

(2) Decomposing and setting up word problems

(3) Symbolic manipulation

(4) Functions

(5) Inductive reasoning and mathematical induction

Milgram (2005)

“A good teacher walks the edge between the structure of mathematics and the development of a child by considering a progression of strategies, the big ideas involved, and the emergent models.”

Fosnot and Jacob, 2010

Arithmetic as a context for algebraic thinking

- The Fundamental Properties of number and operations govern how operations behave and relate to one another
- The Fundamental Properties are essential to computation
- The Fundamental Properties are used more explicitly in some computation strategies than in others
- Simplifying algebraic expressions entails decomposing quantities in insightful ways
- Generalizations in arithmetic can be derived from the fundamental properties.

Arithmetic as a context for algebraic thinking

- "Historically, arithmetic and algebra were treated as distinct fields of study."
- However, a true understanding of arithmetic also includes reasoning about the fundamental properties.
- Generalizations can be formed through exploration:
  - If you add a number to a given number and then subtract that same number, the given number stays the same.

$$a + b – b = a$$

  - An odd number plus an odd number is an even number

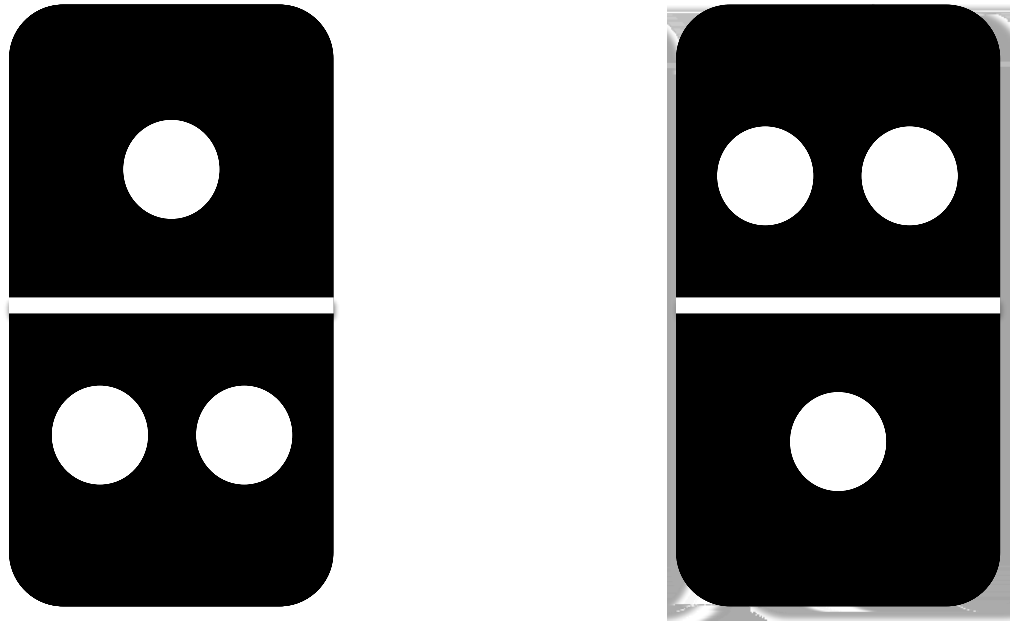
Fundamental Properties

Properties of Addition	• Associative • Commutative • Additive Identity • Additive Inverse
Properties of Multiplication	• Associative • Commutative • Multiplicative Identity • Multiplicative Inverse
Distributive Property of Multiplication over Addition	• Distributive

Use Mini-Lessons

- d 2 5 d 2

Reasoning with Fundamental Properties



$$1 + 2 = 2 + 1$$

Reasoning with Fundamental Properties

9 + 1

9 + 7 + 1

1 + 6 + 9

Reasoning with Fundamental Properties

$$(4 + 9) + 2 \; \boxed{} \; 4 + (9 + 2)$$

$$43 + 17 \; \boxed{} \; 17 + 33$$

$$(568 + 153) + 468 \; \boxed{} \; 658 + (153 + 468)$$

Reasoning with Fundamental Properties

$59 \times 16 \ \square \ 16 \times 15$

$4 \times 5 \ \square \ 5 \times 20$

$(65 \times 2) \times 1 \ \square \ 5 \times (2 \times 1)$

$13\,(15 \times 10) \ \square \ 13 \times 130$

Equations

- The equals sign is a symbol that represents a relationship of equivalence

$$9 + 5 = 8 + 6$$

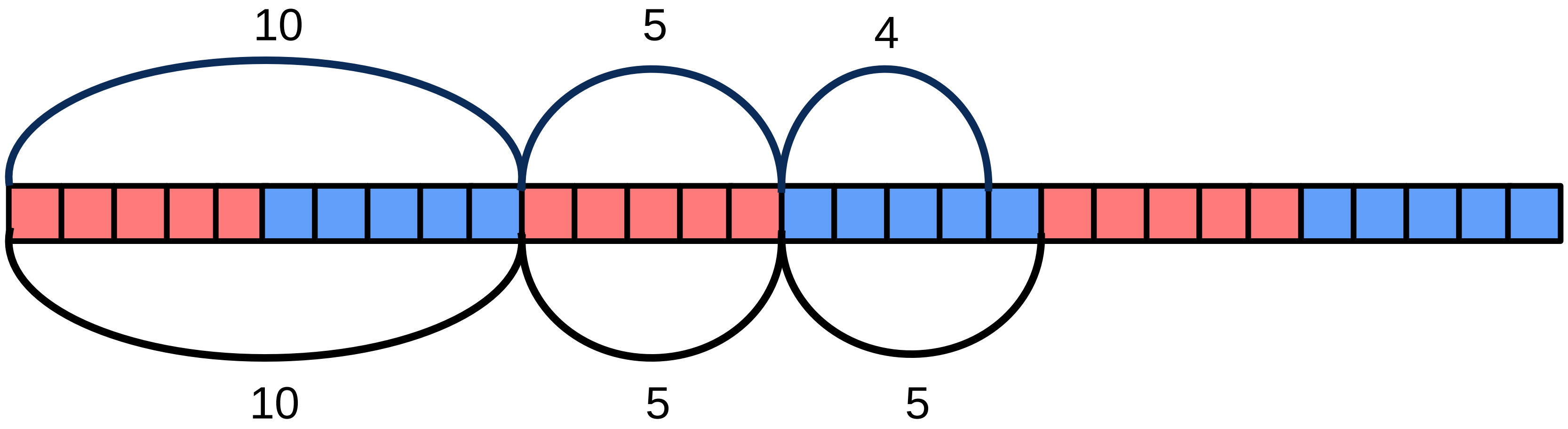
$$13 + 8 + 6 = 5 + 9 + 13$$

$$n + 13 + 9 + 5 = 6 + 8 + 13 + n$$

$$8 + 6 = 5 + 9 + n$$

The double number line

The double number line



Equations

- Equations can be reasoned about in their entirety rather than as a series of computations to execute

Column A	Column B
$48 \times 67 \times 6 = k$	$347 \times 25 \times 4 = k$
$346 \times 398 \div 42 = t$	$398 \times 746 \div 746 = d$
$978 + 778 = 394 + y$	$378 + 794 = 778 + j$
$475 \times 2365 = 352 \times w$	$8790 \times 598 = 879 \times n$

Equations

- Equations can be used to represent problem situations
  - The way we solve a problem does not always match the equation that represents the situation in the problem.

JaeQwan is making flowerpots.
One flowerpot takes ¾ of a pound of clay.
How many flowerpots can JaeQwan make with
4½ pounds of clay?

Representation	Ways to solve
$m \times \frac{3}{4} = 4\frac{1}{2}$	$\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = 4\frac{1}{2}$
	$4\frac{1}{2} \div \frac{3}{4} = 6$

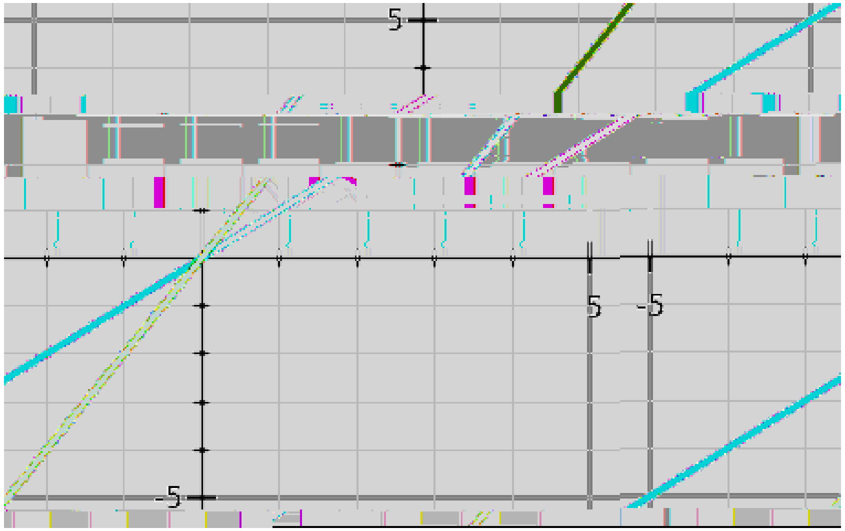
Functional Thinking

$y = 2x$

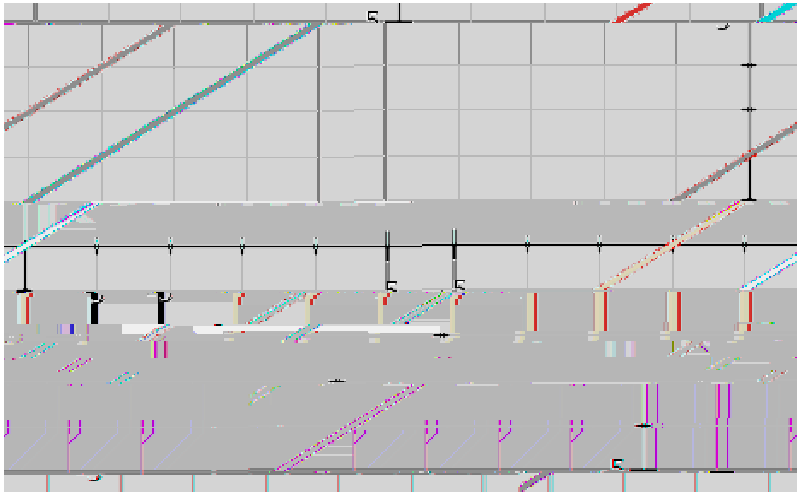
The number of circles is 2 times the position in the pattern.

Functions

$y = 2x$



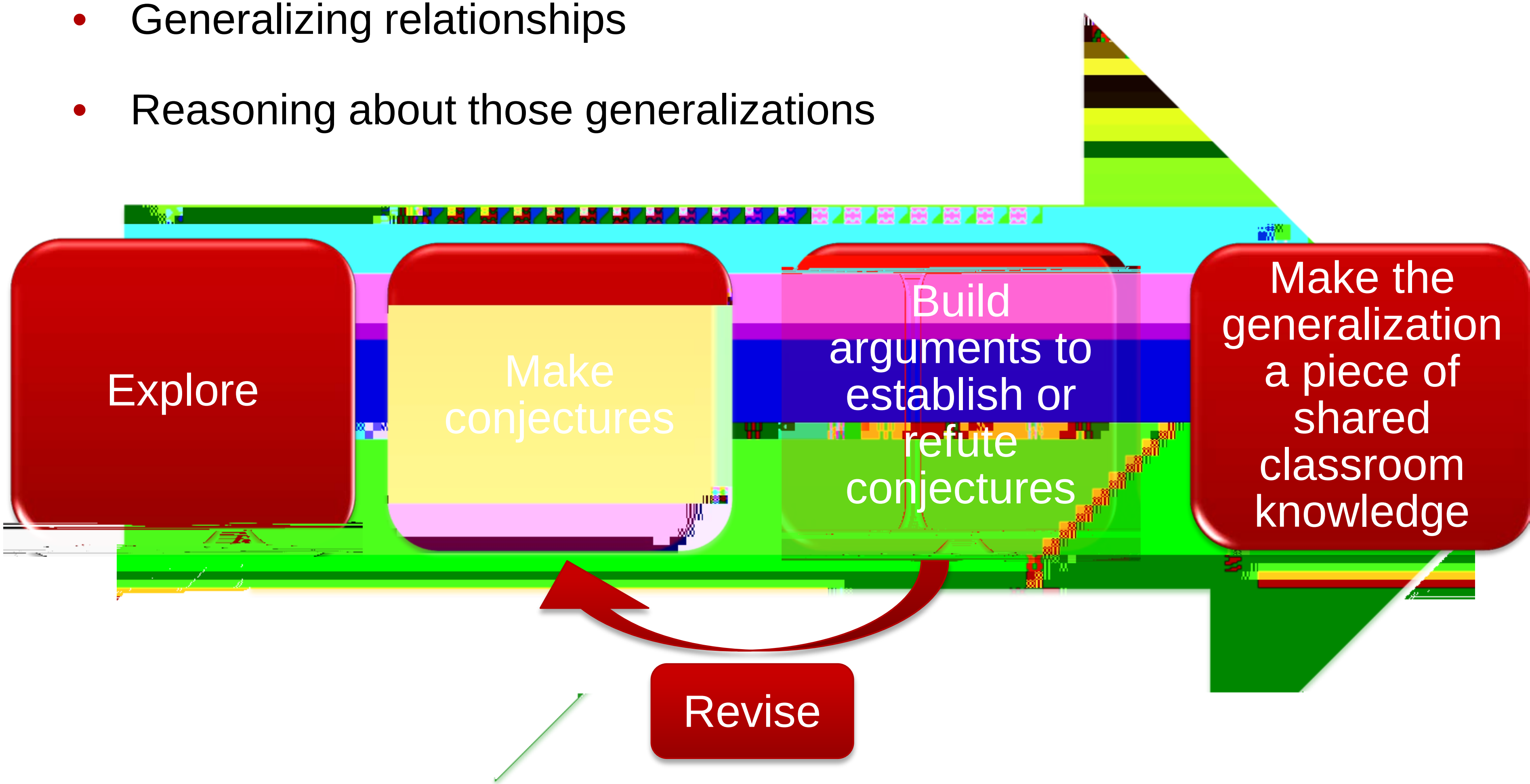
$y = x + 2$



$y = x$

Functional Thinking

- Generalizing relationships
- Reasoning about those generalizations



Blanton et. al, 2011

Kaput et. al, 2008

Teachers are facilitators

"When I'm working on a problem it's like climbing a mountain. Sometimes I can't even see where I'm going. It is one foot in front of another. And then I reach a point where all of a sudden the vistas open up and I can go down easily for a while, only to eventually reach another climb."

Referen .297164s